

AP Calculus AB FRQ Practice: Four Original Teaching Tasks

Worked solutions / self-assessment booklet

Write a complete solution before revealing the worked reasoning. These four independently authored teaching tasks connect sampled data, implicit differentiation, differential equations and an accumulated function. Use the checklists to inspect your methods, conditions and communication, not to calculate an official AP score.

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Scope and review status

Selected AB-level practice for the 2026–27 course. This four-task resource is not a complete six-question exam, an official College Board paper, an official rubric or a validated scoring simulation. Classroom time estimates are editorial. The linked official released questions and scoring guidelines are separate study references.

Author calculations and source-version checks are recorded separately. Independent agent review is distinct from qualified AP educator review. Self-assessment items are local learning criteria, not official marks.

Independent content review: peer checked. Qualified educator review: pending.

This is an original study resource, not an official examination, endorsement, score prediction or certification.

<https://openexam.site/exams/ap-calculus-ab/frq/>

Write, check, and revise

Attempt one task on paper before opening its solutions. Label every subpart, show the setup, and give the reason for each conclusion. A numerical answer alone may hide a missing hypothesis or a sign error.

Use the question-only PDF for a clean attempt, and the separate solution PDF for worked reasoning and self-assessment. These downloads contain the same original tasks as this page.

- Keep approximate results distinct from exact results.
- Name the theorem and verify its conditions when a justification is requested.
- State units in contextual rate and accumulation answers.
- Treat each self-assessment statement as a learning check, not as an official point or AP score.

Four goals and their prerequisites

The tasks emphasize different mathematical decisions. Table values support estimates and a theorem argument; an implicit equation supports tangent and related-rate reasoning; an initial value selects a differential-equation solution; graph vertices support FTC and sign analysis.

Task	Main goal	Suggested time
1	Sampled rates, net accumulation and MVT	18 minutes
2	Implicit derivatives, tangent behavior and related rates	20 minutes
3	Separation, initial values and a justified estimate	18 minutes
4	Read a derivative graph through an accumulation function	18 minutes

How this practice relates to the current exam

The current official exam page lists six free-response questions in 90 minutes: two calculator-required questions in 30 minutes and four calculator-not-permitted questions in 60 minutes. The 2026–27 clarifications do not add new AB course content; the announced May 2027 question-count changes concern multiple choice.

This resource deliberately contains only four teaching tasks. Its practice calculator instructions and suggested times do not recreate a complete official section. The linked released 2026 paper and its scoring guidelines retain their own version-specific instructions.

Continue with official released material

After revising these tasks, open the official released-question index and choose a released paper. Use that paper's own calculator directions and published scoring guidelines. Compare whether your written work establishes the requested setup, reason and interpretation rather than copying an answer.

The numerical data, functions, graph vertices, prompts and local checklists below were authored for this resource. No College Board question or official point table is reproduced, and no qualified educator approval is claimed.

Task 1 — A reservoir observed at uneven times

Topic: Sampled rates, trapezoidal accumulation, net change and the Mean Value Theorem / Suggested time: 18 minutes

Calculator policy: Practice instruction: calculator optional for arithmetic; show the quotient, trapezoidal setup and integral model. These are not official section-placement instructions.

A reservoir contains 60 liters at time $t = 0$. Water enters at a nonnegative rate $R(t)$, in liters per minute, and leaves at the constant rate 4 liters per minute for $0 \leq t \leq 8$. The reservoir has enough capacity for this model. Assume R is continuous on $[0, 8]$ and differentiable on $(0, 8)$. Only the table values of R are known; do not assume linear interpolation. Let $V(t)$ be the amount of water, in liters.

t (minutes)	$R(t)$ (liters/minute)
0	3
2	7
5	5
8	1

Original sampled-rate table. Values between sample times are not specified.

Part a

Estimate $R'(7/2)$ using the average rate of change on $[2, 5]$. Include units and interpret the sign.

1. Use the two specified samples: $R'(7/2) \approx [R(5) - R(2)]/(5 - 2)$.
2. The quotient is $(5 - 7)/3 = -2/3$ liters per minute per minute.
3. The estimate suggests that the inflow rate is decreasing near $t = 7/2$. It is not the rate of change of the reservoir volume, and it is not an exact derivative determined by the table.

Answer: $R'(7/2) \approx -2/3 \text{ L/min}^2$; the inflow rate is estimated to be decreasing.

Self-assessment checks

- Use a difference quotient with both the rate difference and the time difference.
- Report units of liters per minute squared, not liters or liters per minute.
- Identify the derivative as an estimate and interpret its negative sign for the inflow rate.

Part b

Use trapezoids on $[0, 2]$, $[2, 5]$ and $[5, 8]$ to estimate $\int_0^8 R(t) dt$. Explain what this integral measures.

1. The interval widths are 2, 3 and 3 minutes; the partition is not uniform.

2. The trapezoidal estimate is $2(3 + 7)/2 + 3(7 + 5)/2 + 3(5 + 1)/2$.

3. The three contributions are 10, 18 and 9 liters, so the estimate is 37 liters.

4. The integral measures the total water entering during the eight-minute interval. Since the true rates between samples are unspecified, the trapezoidal result is an estimate, not a guaranteed exact total.

Answer: $\int_0^8 R(t) dt \approx 37$ liters of incoming water.

Self-assessment checks

- Use all three actual interval widths with the average of each pair of endpoint rates.
- Obtain 37 from the displayed trapezoidal setup.
- Interpret incoming volume over the stated interval and preserve the approximation sign.

Part c

Write an exact integral expression for $V(8)$, then estimate $V(8)$ using part (b).

1. The net rate of volume change is $V'(t) = R(t) - 4$.

2. Initial amount plus accumulated net change gives $V(8) = 60 + \int_0^8 [R(t) - 4] dt$.

3. The constant outflow removes $4 \cdot 8 = 32$ liters. Use the estimate 37 liters for inflow:
 $V(8) \approx 60 + 37 - 32 = 65$ liters.

Answer: Exact expression: $60 + \int_0^8 [R(t) - 4] dt$. Estimated amount: 65 liters.

Self-assessment checks

- Include the initial 60 liters.
- Subtract the outflow over the full eight minutes rather than subtracting 4 only once.
- Distinguish the exact integral model from its 65-liter numerical estimate.

Part d

Prove that some $c \in (2, 5)$ satisfies $R'(c) = -2/3$. Does this prove that $R'(7/2) = -2/3$ exactly? Explain.

1. The assumed continuity on $[0, 8]$ implies continuity on $[2, 5]$, and differentiability on $(0, 8)$ implies differentiability on $(2, 5)$.

2. The Mean Value Theorem therefore provides some $c \in (2, 5)$ with $R'(c) = [R(5) - R(2)]/(5 - 2) = -2/3$.

3. The theorem guarantees an unspecified interior point. It does not identify c as $7/2$, so it does not convert part (a)'s estimate at that particular point into an exact derivative.

Answer: Such a c exists by the Mean Value Theorem, but its location need not be $7/2$.

Self-assessment checks

- Verify continuity on the closed interval and differentiability on its interior.
- Use the endpoint quotient to establish the guaranteed derivative value.
- Separate existence of some interior point from identifying the midpoint.

Task 2 — A positive branch of an implicit constraint

Topic: Implicit derivatives, tangent approximation, concavity and related rates / Suggested time: 20 minutes

Calculator policy: Practice instruction: no calculator is needed. Keep exact fractions; show implicit differentiation before substituting the point.

Consider the curve $2x^2 + xy + 3y^2 = 13$ and its positive, twice-differentiable branch through $(2, 1)$. This branch remains positive for $2 \leq x \leq 21/10$. Parts(a)–(c) use dimensionless coordinates; part (d) introduces a separate length-and-time model with the same algebraic constraint.

Part a

Find dy/dx in terms of x and y . Then find the tangent-line equation at $(2, 1)$.

1. Differentiate every term with respect to x : $4x + y + xy' + 6yy' = 0$. The xy term requires the product rule.
2. Collect the derivative terms: $(x + 6y)y' = -(4x + y)$, so $y' = -(4x + y)/(x + 6y)$.
3. At $(2, 1)$, the denominator is 8 and the numerator is 9, so the slope is $-9/8$.
4. The tangent line is $L(x) = 1 - (9/8)(x - 2)$.

Answer: $y' = -(4x + y)/(x + 6y)$; $y - 1 = -(9/8)(x - 2)$.

Self-assessment checks

- Differentiate xy with the product rule and y^2 with the chain rule.
- Collect y' terms before dividing, and note that $x+6y$ is nonzero at the point.
- Use both the slope $-9/8$ and the point $(2, 1)$ in the tangent equation.

Part b

Find d^2y/dx^2 at $(2, 1)$. Justify the sign of the result.

1. Differentiate $4x + y + (x + 6y)y' = 0$ once more: $4 + 2y' + 6(y')^2 + (x + 6y)y'' = 0$.
2. Therefore $y'' = -[4 + 2y' + 6(y')^2]/(x + 6y)$.
3. Substitute $y' = -9/8, x = 2, y = 1$. The numerator is $4 - 18/8 + 6(81/64) = 299/32$; divide by 8 with the leading minus sign.
4. Thus $y''(2) = -299/256 < 0$, which means this branch is concave down at the point.

Answer: $y''(2) = -299/256$, so the branch is concave down there.

Self-assessment checks

- Account for both y' contributions and the $6(y')^2$ term.
- Use the previously found slope and compute the exact fraction $-299/256$.
- Link the negative second derivative to concavity rather than to whether y itself is positive.

Part c

Use the tangent line to estimate y at $x = 21/10$. Is the tangent estimate above or below the actual branch value? Justify the comparison throughout $[2, 21/10]$, not just by giving the sign at one point.

1. Insert $x - 2 = 1/10$ into the tangent line: $L(21/10) = 1 - 9/80 = 71/80$.
2. On the stated positive branch, $x + 6y > 0$. For any real value $p = y'$,
 $4 + 2p + 6p^2 = 6(p + 1/6)^2 + 23/6 > 0$.
3. The second-derivative expression from part (b) is therefore negative throughout the stated interval. The branch is concave down there, so its tangent line lies above the branch.
4. The estimate $71/80$ is an overestimate. A negative second derivative only at the initial point would not, by itself, justify the comparison over the entire requested interval.

Answer: $y(21/10) \approx 71/80 = 0.8875$, an overestimate.

Self-assessment checks

- Evaluate the tangent line at the requested input rather than treating the slope as the estimate.
- Use positivity of the denominator and strict positivity of the quadratic numerator on the whole interval.
- Conclude that concave-down behavior puts this tangent estimate above the branch.

Part d

In a separate model, x and y are positive lengths in centimeters that depend on time s , in seconds, and satisfy the same constraint (with constant 13 square centimeters). At the instant $x = 2, y = 1$ and $dx/ds = 4$ centimeters per second, find dy/ds . Interpret the sign.

1. Differentiate the constraint with respect to time: $(4x + y) dx/ds + (x + 6y) dy/ds = 0$.
2. At the stated instant, $9 \cdot 4 + 8 dy/ds = 0$.
3. Hence $dy/ds = -36/8 = -9/2$ centimeters per second. The negative sign means the length y is decreasing at that instant.

Answer: $dy/ds = -9/2$ cm/s; the length y is decreasing.

Self-assessment checks

- Differentiate both changing lengths with respect to the same time variable.
- Use 9 and 8 for the two coefficient values at the instant.
- Give the negative rate $-9/2$ with units and its contextual interpretation.

Task 3 — Select and test a differential-equation solution

Topic: Separation of variables, initial conditions, concavity and a unique threshold / Suggested time: 18 minutes

Calculator policy: Practice instruction: exact work requires no calculator. A calculator may be used only to report the optional three-decimal value in part (d).

A positive function y satisfies $dy/dx = (2x + 1)/(2y)$ for $0 \leq x \leq 2$, with $y(0) = 2$. The variables in this task are dimensionless. Write supporting differentiation or integration steps, not just a candidate formula.

Part a

Find $y'(0)$, write the tangent line at $x = 0$, and use it to estimate $y(1/5)$.

1. The differential equation gives $y'(0) = 1/(2 \cdot 2) = 1/4$.
2. The line through $(0, 2)$ with this slope is $L(x) = 2 + x/4$.
3. Therefore $y(1/5) \approx L(1/5) = 2 + 1/20 = 41/20$.

Answer: $y'(0) = 1/4$; $L(x) = 2 + x/4$; $y(1/5) \approx 41/20$.

Self-assessment checks

- Substitute both $x=0$ and $y=2$ into the differential equation.
- Use the initial value as the tangent-line intercept.
- Label $41/20$ as an approximation rather than an exact solution value.

Part b

Use separation of variables to find the particular solution. Explain how the sign of the square root is determined, and verify the differential equation and initial condition.

1. Because the stated solution is positive, its denominator is nonzero. Separate and integrate: $2y \, dy = (2x + 1) \, dx$, so $y^2 = x^2 + x + C$.
2. The initial condition gives $4 = C$. Thus $y = \sqrt{x^2 + x + 4}$; the negative square-root branch would have value -2 at $x=0$ and violates both the initial condition and positivity.
3. Differentiating the positive branch yields $y' = (2x + 1)/(2\sqrt{x^2 + x + 4}) = (2x + 1)/(2y)$. Also $y(0) = \sqrt{4} = 2$.

Answer: $y(x) = \sqrt{x^2 + x + 4}$ on the stated interval.

Self-assessment checks

- Show separation and integration with a constant of integration.
- Determine $C=4$ and explicitly choose the positive branch.
- Verify both the derivative equation and the initial value; one check alone is incomplete.

Part c

Use the particular solution to prove that the tangent estimate in part (a) is an underestimate.

1. Differentiate the first-derivative expression, using y as a function of x : $y'' = [4y^2 - (2x + 1)^2]/(4y^3)$.
2. From part (b), $y^2 = x^2 + x + 4$. Hence the numerator is $4(x^2 + x + 4) - (2x + 1)^2 = 15$.
3. Therefore $y'' = 15/(4y^3) > 0$ on $[0, 2]$. The graph is concave up, so the tangent line at 0 lies below it for $0 < x \leq 2$, including $x = 1/5$.

Answer: $41/20 < y(1/5)$; the tangent estimate is an underestimate.

Self-assessment checks

- Differentiate with the chain/quotient rules rather than treating y as constant.
- Show the numerator simplifies to 15 and use $y > 0$.
- Use concavity over the relevant interval to justify the tangent comparison.

Part d

Find the first $x \in [0, 2]$ for which $y(x) = 3$. Give an exact answer; an optional decimal should be rounded to three places.

1. Set $\sqrt{x^2 + x + 4} = 3$. Squaring gives $x^2 + x - 5 = 0$.
2. The two algebraic roots are $(-1 \pm \sqrt{21})/2$. Only $(-1 + \sqrt{21})/2$ lies in $[0, 2]$.
3. On this interval, $2x + 1 > 0$ and $y > 0$, so $y' > 0$. The solution is strictly increasing, which makes this crossing unique and therefore the first one.
4. The optional decimal is 1.791.

Answer: $x = (-1 + \sqrt{21})/2 \approx 1.791$, the unique crossing in the stated interval.

Self-assessment checks

- Solve the threshold equation rather than using the tangent approximation as the actual solution.
- Discard the negative root using the stated interval.
- Justify that the positive crossing is unique by the sign of y' .

Task 4 — Recover behavior from a graph of a rate

Topic: FTC, signed geometric area, local extrema and absolute extrema / Suggested time: 18 minutes

Calculator policy: Practice instruction: no calculator is needed. Use signed areas of triangles, trapezoids and rectangles and show the endpoint comparisons.

A continuous function h on $[-2, 6]$ has a graph consisting exactly of straight-line segments joining consecutive vertices in the table. The table is an original exact description of the graph, not a collection of unspecified samples. Define $G(x) = -2 + \int_0^x h(t) dt$. All coordinates and function values in this task are dimensionless.

x-coordinate	$h(x)$ at a graph vertex
-2	1
0	-1
2	-1
4	3
6	1

Original polygonal graph represented by its vertices. Join consecutive rows by straight segments; between $x=0$ and $x=2$ the graph is horizontal.

Part a

Find $G'(3)$ and $G''(3)$. Explain how each value is read from the given graph.

1. Since h is continuous, the Fundamental Theorem of Calculus gives $G'(x) = h(x)$.
2. Between 2 and 4, the line through $(2, -1)$ and $(4, 3)$ has slope $[3 - (-1)]/(4 - 2) = 2$, so $h(3) = 1$.
3. Therefore $G'(3) = 1$. At this non-corner point, $G''(3) = h'(3) = 2$. The value of h and the slope of h answer different derivative questions.

Answer: $G'(3) = 1$ and $G''(3) = 2$.

Self-assessment checks

- Use $G'=h$ from continuity and FTC.
- Read the line value 1 at $x=3$ rather than using its slope for G' .
- Use the segment slope 2 for G'' and note that $x=3$ is not a corner.

Part b

Compute $G(4)$ using signed areas.

1. On $[0, 2]$, $h=-1$, so $\int_0^2 h(t) dt = -2$.

2. On $[\tilde{2}, 4]$, the exact signed trapezoidal area is $2[-1 + 3]/2 = 2$. Equivalently, the small negative triangle has area $-1/4$ and the positive triangle has area $9/4$.
3. Thus $\int_0^4 h(t) dt = 0$, and $G(4) = -2 + 0 = -2$.

Answer: $G(4) = -2$.

Self-assessment checks

- Use signed area, not total unsigned area.
- Include the constant -2 in G .
- Explain that the integral is exact here because the complete polygonal graph is specified.

Part c

Find every interior local maximum and local minimum of G . Give the x -values and justify each classification.

1. The segment from $(-2, 1)$ to $(0, -1)$ crosses the x -axis at -1 . The segment from $(2, -1)$ to $(4, 3)$ crosses at $5/2$.
2. Because $G' = h$, G is increasing on $(-2, -1)$, decreasing on $(-1, 5/2)$, and increasing on $(5/2, 6)$.
3. At -1 , the derivative changes from positive to negative, giving a local maximum. At $5/2$, it changes from negative to positive, giving a local minimum. The other graph vertices are not zeros of h and do not create additional local extrema of G .

Answer: Interior local maximum at $x = -1$; interior local minimum at $x = 5/2$.

Self-assessment checks

- Find zeros of h rather than zeros or corners of G without evidence.
- State the derivative sign change at each critical point.
- Check all interior possibilities rather than identifying only one extremum.

Part d

Find the absolute maximum and absolute minimum values of G on $[-2, 6]$, and where they occur. Justify why the candidate list is complete.

1. G is continuous on the closed interval because h is continuous. The Extreme Value Theorem guarantees absolute extrema. The endpoint/critical-point candidates are $-2, -1, 5/2, 6$, since $G' = h$ exists throughout the interior and is zero only at -1 and $5/2$.
2. The signed areas on $[-2, 0]$ cancel, so $G(-2) = -2$. On $[-1, 0]$, the integral of h is $-1/2$; reversing the integral gives $G(-1) = -2 + 1/2 = -3/2$.
3. From 0 to 2 , the integral is -2 ; from 2 to $5/2$, it is a negative triangle of area $-1/4$. Hence $G(5/2) = -2 - 2 - 1/4 = -17/4$.
4. Part(b) gives $G(4) = -2$. The exact trapezoidal area on $[4, 6]$ is $2(3 + 1)/2 = 4$, so $G(6) = 2$.
5. Compare all four candidate values: the absolute minimum is $-17/4$ at $x = 5/2$, and the absolute maximum is 2 at $x = 6$.

Answer: Absolute minimum $-17/4$ at $x = 5/2$; absolute maximum 2 at $x = 6$.

Self-assessment checks

- Use continuity for existence, but also identify the complete endpoint/critical-point candidate list.
- Handle the reversed integral for negative inputs with the correct sign.
- Compute and compare all four candidate values; local extrema alone do not establish absolute extrema.

REFERENCE TRAIL

Sources, version and scope

The linked materials are references for the independently authored explanation and task design. Public access does not imply permission to reproduce an official examination bank.

AP Calculus AB course and current content framework

<https://apcentral.collegeboard.org/courses/ap-calculus-ab>

Reference version: 2026–27 course page, reviewed 2026-10-28

Locator: Course Content and Mathematical Practices; eight-unit AB framework.

AP Calculus AB exam overview

<https://apcentral.collegeboard.org/courses/ap-calculus-ab/exam>

Reference version: Current 2026–27 page / May 2027 exam, reviewed 2026-10-28

Locator: Clarifications and Exam Updates for 2026–27; SectionII:Free Response.

AP Calculus AB and BC Course and Exam Description

<https://apcentral.collegeboard.org/media/pdf/ap-calculus-ab-and-bc-course-and-exam-description.pdf>

Reference version: CED linked by the current 2026–27 course page; retained 2026-10-28

Locator: Topics 3.2,3.6,4.4–4.6,5.1–5.6,6.2,6.4–6.5, and 7.6–7.7. Relevant PDF pages 76,80,90–92,99–104,119,121–122,142–143.

AP Calculus AB and BC CED clarifications and corrections

<https://apcentral.collegeboard.org/media/pdf/ap-calculus-ab-bc-course-and-exam-description-clarifications.pdf>

Reference version: Effective Fall 2026

Locator: PDF pages 1–2:EVT clarification, FUN-7. B.2, and May 2027 MCQ changes.

Official AP Calculus AB released FRQs and scoring resources

<https://apcentral.collegeboard.org/courses/ap-calculus-ab/exam/past-exam-questions>

Reference version: Current public index, reviewed 2026-10-28

Locator: 2026 released FRQ and Scoring Guidelines links.

2026 AP Calculus AB released free-response questions

<https://apcentral.collegeboard.org/media/pdf/ap26-frq-calculus-ab.pdf>

Reference version: 2026 released examination

Locator: PDF pages 2 and 6:calculator/noncalculator directions and written-justification instructions. Questions are linked for separate practice, not reproduced here.

2026 AP Calculus AB released scoring guidelines

<https://apcentral.collegeboard.org/media/pdf/ap26-sg-calculus-ab.pdf>

Reference version: 2026 released scoring guidelines

Locator: PDF page 2:general scoring notes and setup/units distinction; question-specific notes illustrate that criteria are tied to actual official prompts. No official point scheme is assigned to these new tasks.